**Math 245**

**Spring 2022**

**Lecture 22**

as $x \to -\infty$
$f(x) \to \infty$

as $x \to \infty$
$f(x) \to -\infty$

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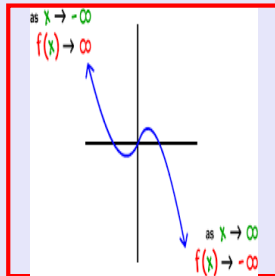
Complex Numbers:

Standard Form:  $a + bi$  $i = \sqrt{-1}$

$i^2 = -1$

Real Part

Imaginary Part

$3 - 2i$    Re. Part $= 3$

Im. Part $= -2$

$\dfrac{-3}{5} + \dfrac{4}{5}i$    Re. $= \dfrac{-3}{5}$ , Im. $= \dfrac{4}{5}$

Abs. Value of complex number $z = a + bi$

$$|z| = \sqrt{a^2 + b^2}$$

Suppose  $z = -4 + 3i$

$$|z| = \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = \boxed{5}$$

Given $Z = 6 - 8i$

$Z = a + bi$

1) Re. Part

$\boxed{a = 6}$

2) Im. Part

$\boxed{b = -8}$

3) Find $|Z| = \sqrt{a^2 + b^2}$

$= \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = \boxed{10}$

Express $\sqrt{-36}$ in Complex Form

$\sqrt{-36} = \sqrt{36}\sqrt{-1} = 6i$

Express $\sqrt{25} - \sqrt{-100}$ in Complex Form.

$\sqrt{25} - \sqrt{-100} = 5 - \sqrt{100}\sqrt{-1} = \boxed{5 - 10i}$

Re. 5
Im. -10

Express $3\sqrt{40} - 2\sqrt{-72}$ in Complex Form.

$3\sqrt{40} - 2\sqrt{-72} = 3\sqrt{4}\sqrt{10} - 2\sqrt{36}\sqrt{2}\sqrt{-1}$

$= 3 \cdot 2\sqrt{10} - 2 \cdot 6\sqrt{2}\, i$

$= 6\sqrt{10} - 12\sqrt{2}\, i$

Re. $6\sqrt{10}$

Im. $-12\sqrt{2}$

$\boxed{= 6\sqrt{10} - 12i\sqrt{2}}$

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Powers of $i$:

1) Even Powers

$i^{100} = (i^2)^{50}$

$= (-1)^{50} = \boxed{1}$

$i^{30} = (i^2)^{15}$

$= (-1)^{15} = \boxed{-1}$

Recall

$(x^m)^n = x^{mn}$

$i = \sqrt{-1}$

$i^2 = -1$

$(-1)^{even} = 1$

$(-1)^{odd} = -1$

Simplify

1) $i^{150} = (i^2)^{75}$

$= (-1)^{75}$

$= \boxed{-1}$

2) $i^{60} = (i^2)^{30}$

$= (-1)^{30}$

$= \boxed{1}$

Powers of $i$:

2) Odd Powers

$$i^{21} = i^{20} \cdot i$$
$$= (i^2)^{10} \cdot i$$
$$= (-1)^{10} \cdot i$$
$$= 1 \cdot i = \boxed{i}$$

$$i^{51} = i^{50} \cdot i = (i^2)^{25} \cdot i = (-1)^{25} \cdot i = -1 \cdot i$$
$$= \boxed{-i}$$

Recall
$$(x^m)^n = x^{mn}$$
$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$(-1)^{even} = 1$$
$$(-1)^{odd} = -1$$
$$x^m \cdot x^n = x^{m+n}$$

Simplify: $i^{75} = i^{74} \cdot i$
$$= (i^2)^{37} \cdot i = (-1)^{37} \cdot i = -1 \cdot i = \boxed{-i}$$

Simplify $i^{93}$

$$i^{93} = i^{92} \cdot i = (i^2)^{46} i = (-1)^{46} \cdot i = 1 \cdot i = \boxed{i}$$

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Distribute & Simplify

$$2(3 - 4i) - 5(4 - 2i)$$

$$= 6 - 8i - 20 + 10i$$
$$= \boxed{-14 + 2i} \quad \begin{array}{l} Re. \ -14 \\ Im. \ 2 \end{array}$$

Distribute & Simplify
$$3i(1 - 2i) - 6(1 + 2i)$$

$$= 3i \cdot 1 - 3i \cdot 2i - 6 \cdot 1 - 6 \cdot 2i$$
$$= 3i - 6i^2 - 6 - 12i$$
$$= 3i - 6(-1) - 6 - 12i$$
$$= -9i + 6 - 6 = \boxed{-9i} \quad \begin{array}{l} Re. \ 0 \\ Im. \ -9 \end{array}$$

Foil & Simplify

$(2+3i)(2-3i)$

$= 4 - 6i + 6i - 9i^2$

$= 4 - 9(-1) = 4 + 9 = \boxed{13}$  Re. 13  Im. 0

Foil & Simplify

$(4-3i)(5+2i)$

$= 20 + 8i - 15i - 6i^2$

$= 20 - 7i - 6(-1)$

$= 20 - 7i + 6 = \boxed{26 - 7i}$  Re. 26  Im. -7

Simplify  $(2+5i)^2$

Hint:  $x^2 = x \cdot x$

$= (2+5i)(2+5i)$

$= 4 + 10i + 10i + 25i^2$

$= 4 + 20i + 25(-1)$

$= 4 + 20i - 25$

WORK on SG 9

$= \boxed{-21 + 20i}$  Re. -21  Im. 20